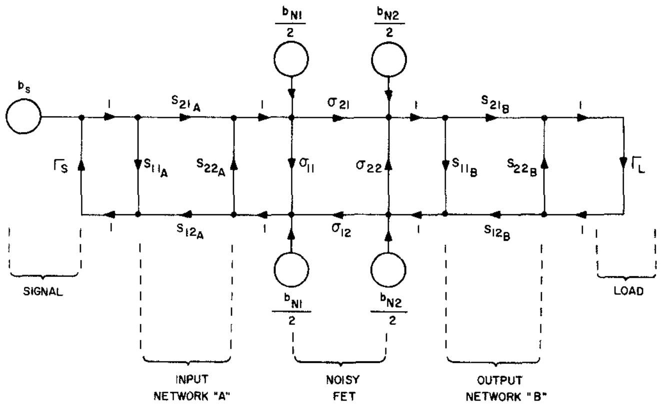

Fig. 3. Signal flow graph equivalent to the noisy FET of Fig. 1, embedded between input and output networks which provide dc bias as well as impedance transformation The noise sources b_{N1} and b_{N2} are the S-parameter equivalents of the noise current sources i_g and i_d of the FET model of Fig. 1. The σ's represent the four S-parameters of the FET.

moderate frequencies with signal flow graph equivalents, so that the noise performance of the device can be analyzed in terms of its S-parameters. Fig. 3, for example, shows the signal flow graph which results when a field-effect transistor, whose moderate-frequency noise model is that devised by van der Ziel (Fig. 1), is embedded between input and output linear networks serving to provide dc bias to the device as well as performing the function of transforming impedances. The source b_S in Fig. 3 represents a *signal* source, not a noise source, and is defined in the usual manner [2]. That is to say

$$b_S = \frac{V_S\sqrt{Z_0}}{Z_S + Z_0}, \qquad \Gamma_s = \frac{Z_S - Z_0}{Z_S + Z_0}. \tag{11}$$

Since, at microwave frequencies, the "natural" measurable parameters of a solid-state device, as well as those of the linear networks in which it is embedded, are the S-parameters, it seems to the authors that the "natural" manner in which to characterize the noise sources of the device is that which has been developed in this short paper. In a sequel paper, we shall describe the results of experiments with a GaAs MESFET at 3 GHz still in progress which have as their goal the direct measurement of the noises $\langle b_{N1}b_{N1}^*\rangle$, $\langle b_{N2}b_{N2}^*\rangle$, and their cross-correlation $\langle b_{N1}b_{N2}^*\rangle$ (both real and imaginary parts) of the sources shown in Fig. 3.

V. Addendum

While the above paper was under second review following revision, Hecken [3] published an important paper also dealing with the use of S-parameters to characterize the noise of active devices. A brief comment on how his approach differs from the above is in order. Whereas our equivalent noise sources relate directly to the current and/or voltage sources encountered in moderate frequency models for device noise, Hecken simply includes sources contributing to the "b"-waves emanating from each port of the device. There is nothing wrong with that approach. However, it cannot be used to interpret the physical mechanisms accounting for the noise. Inspection of Fig. 3, for example, reveals that both b_{N1} and b_{N2} (consequently van der Ziel's current sources i_g and i_d in Fig. 1) contribute to the noise emanating from both ports of the noisy FET, even if those ports see perfect impendance matches connected to them. In other words, Hecken's B_{q1} and B_{q2} are due to an admixture of the contributions from two differing physical mechanisms for the noise, and measurement of them in the neat way that he describes will provide no physical insight as to their cause because, with both ports matched, only his noise source B_{q2} contributes to the noise emanating from port 2 and vice-versa.

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On the Inherent Noise of an Ideal Two-Port Isolator

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Abstract—**The inherent noise of an ideal isolator, predicted by Siegman from a simple thermodynamic argument, is verified by examining in detail the behavior of a common two-port isolator configuration, insofar as its sources of noise are concerned. Siegman's conclusion that any ideal isolator is the equivalent of a terminated three-port circulator is vindicated.**

I. Introduction

In 1961 Siegman [1] concluded from a thermodynamic argument that all isolators are the equivalent of terminated circulators insofar as their inherent noise is concerned. That argument is so neat that it is worth reviewing, especially since it appeared in a 20-year-old journal not likely to be available to the reader. Fig. 1(a) shows an ideal isolator which transmits a signal from port 1 to port 2 without attenuation, but attenuates perfectly any signal incident upon port 2 without reflection. Siegman described the following paradox. Suppose port 1 is matched with a termination at a *cold* temperature T_1 while port 2 is similarly matched at a *hot* temperature T_2. Then *all* of the Nyquist noise power $kT_1\Delta f$ from the cold resistor at port 1 reaches and is absorbed by the matched load at port 2, but *none* of the Nyquist noise power $kT_2\Delta f$ from the hot resistor at port 2 reaches its counterpart at port 1. This is clearly a violation of the second law of thermodynamics since we have a continuous transfer of heat from a cold source to a hot sink. Based on this paradox, Siegman argued that an ideal isolator must be modeled as an ideal three-port circulator with its port 3 resistively matched, as shown in Fig. 1(b), that termination accounting for the dissipative losses *inherent* in an isolator in order to produce its nonreciprocal action. This clearly removes the paradox because now the additional Nyquist noise power $kT_3\Delta f$ exits port 1 and is absorbed by the cold termination there. A signal flow analysis of the latter situation reveals that the *net* noise power entering all three ports sums to zero, and the second law is satisfied.

One concludes from the above argument that the inherent thermal noise of any isolator is Nyquist noise $kT\Delta f$ exiting from port 1 only. That this is indeed the case is not intuitively obvious

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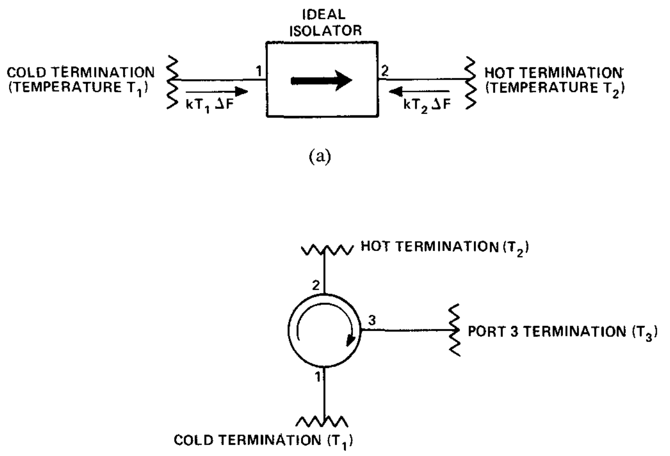

(b)

Fig. 1. (a) Siegman's thermodynamic conundrum for an ideal isolator (b) The terminated ideal 3-port circulator which solves the conundrum.

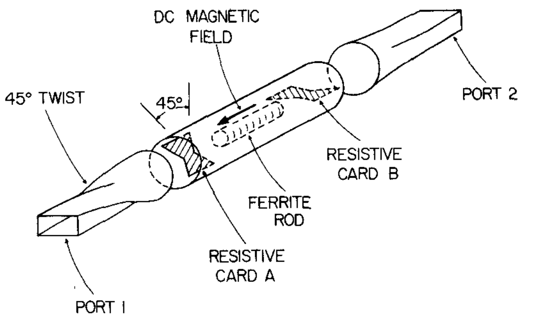

Fig. 2. A schematic representation of a two-port Faraday rotation isolator.

for a common embodiment of an isolator shown in Fig. 2. It is the purpose of this short paper to demonstrate that Siegman's conclusion is correct and that there are some subtleties concerning just *what* is the source of that noise.

II. Analysis

The isolator in Fig. 2 consists of a length of circular waveguide which is above cutoff only for its dominant TE_{11}-mode and which contains a ferrite rod, biased by an appropriate axial dc magnetic field, causing a 45° Faraday rotation of that mode pattern. Two circular-to-rectangular waveguide transitions, one introducing a 45° twist, connect to each end. Thin resistive cards, located within the circular section as shown, are oriented so as to couple to the TE_{01}-mode of the rectangular sections, which is below cutoff and therefore evanescent.

Fig. 3 shows the electric field orientation at five cross sections of the device: 1) port 1, 2) resistive card A, 3) ferrite rod, 4) resistive card B, 5) port 2. (Actually 3) does not represent a "cross section," but rather shows the Faraday rotation introduced by the ferrite rod between the two resistive cards.) The figure depicts four cases:

(a) TE_{10}-mode incident upon port 1 from the left;
(b) TE_{10}-mode incident upon port 2 from the right;
(c) TE_{01}-mode incident upon port 1 from the left; and
(d) TE_{01}-mode incident upon port 2 from the right.

Cases (c) and (d) are included to recognize that, *because* the TE_{01}-mode is below cutoff, its wave impedance is imaginary so that any energy exiting port 1 in that mode is totally reflected, case (c), and similarly for any energy exiting port 2 in that mode, case (d).

Examining Fig. 3(a), we see that any signal entering port 1

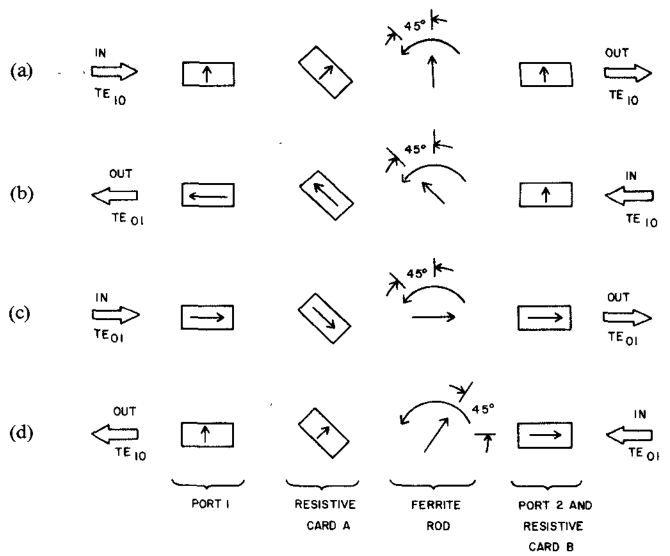

Fig. 3. Sketches of the E-field vector at five cross sections of the isolator of Fig 2 for four differing cases. (a) TE_{10} wave incident from the left upon port 1. (b) TE_{10} wave incident from the right upon port 2. (c) TE_{01} wave incident from the left upon port 1. (d) TE_{01} wave incident from the right upon port 2.

from an external source "upstream" of it (said signal *must* be in the TE_{10}-mode to have reached port 1) exits port 2 unattenuated assuming an ideal device, i.e., no waveguide loss, a lossless ferrite, and an infinitesimal thickness of the resistive cards. Fig. 3(b) shows that any signal entering port 2 from an external source "downstream" of it is converted into the TE_{01}-mode which is absorbed by resistive card A, so that no signal exits port 1. The above is the normal isolator action.

Now consider the noise generated by the two resistive cards. Card A excites waves propagating away from it in both directions, each carrying Nyquist noise power $kT\Delta f$ (see [1]). The wave exiting port 1 in the TE_{01} (rectangular)-mode, being evanescent, is totally reflected, reentering that port and being reabsorbed by the resistive card which was its source. The wave exiting that card to the right in the TE_{11} (circular)-mode is rotated so as to be absorbed by resistive card B, as seen in Fig. 3(c).

Fig. 3(d) reveals that the noise culprit is resistive card B. It, too, excites waves propagating in both directions, each carrying Nyquist noise $kT\Delta f$. That exiting port 2 is reflected and reabsorbed just as above. But, as Fig. 3(d) reveals, the TE_{11} (circular)-mode of the wave propagating to the left is rotated so as to emerge from port 1 in the dominant TE_{10} (rectangular)-mode. We conclude that *only* port 1 exhibits noise generated thermally within this device and that the noise exiting that port is indeed the Nyquist noise $kT\Delta f$ which would result from an ideal circulator whose port 3 is resistively matched at temperature T. Siegman's contention is correct.

One can play "games" with Fig. 3. For example, one might ask what the noise result would be if the obvious "culprit", namely resistive card B, were eliminated. Reinspection of Figs. 3(a) and 3(b) reveals that this should have no effect upon the operation of the device as an isolator since, in neither case, would the presence or absence of that card be detected by the TE_{10}-modes exiting or entering port 2. But Fig. 3(c) now reveals that the Nyquist noise emanating to the right from the remaining resistive card A, emerging unchanged as a TE_{01}-mode from port 2, being totally reflected and reentering said port, would then be transformed by the Faraday rotator into the dominant TE_{10}-mode exiting port 1 as Fig. 3(d) reveals, again producing Nyquist noise $kT\Delta f$ emanating from that port as above. Thermodynamics triumphs!

Of course, if both resistive cards are eliminated, we have no isolator at all. Examination of Figs. 3(b), (c), and (d) reveals that a TE_{10} signal incident from the right at port 2 exits port 1 in the TE_{01}-mode, is reflected, exits port 2 in the same mode, is again reflected, and ultimately exits port 1 unimpeded in the TE_{10}-mode.

III. Discussion

Our motivation for examining the inherent noise of isolators came from the fact that we wished to use them in a noise measurement system at 94 GHz, with the goal of detecting the quantum noise of devices at 2 K. Some details of the measurement system envisioned are given in a companion paper [2]. It became obvious that the Nyquist noise due to the termination of port 3 of a circulator used as an isolator would be intolerable for our purpose. Naively, we thought that a Faraday rotation isolator embodied in the form of Fig. 2 might behave differently. We then undertook the analysis given above and discovered, much to our surprise, that it behaved exactly as a terminated circulator. Only then did we discover Siegman's incontrovertible and beautifully simple proof of that fact.

IV. Conclusion

To achieve isolation between its input and output ports, an isolator *must* include at least one resistive source of Nyquist noise. That noise emanates from its input port. Siegman's thermodynamic proof cannot be denied.

Acknowledgment

This work was supported by National Science Foundation Grant ECS-8007623. A. van der Ziel suggested to the author that the inherent noise of an isolator required exploration.

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The Exact Noise Figure of Amplifiers with Parallel Feedback and Lossy Matching Circuits

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Abstract —**Exact formulas for the noise parameters and noise figure of amplifiers with parallel feedback and lossy input and output matching circuits are derived. The formulas which take into account the thermal agitation of all circuit elements are applicable to feedback and lossy match amplifiers, as well as amplifiers that employ both principles simultaneously.**

I. Introduction

Recent developments in the design of single-ended GaAs MESFET amplifiers have focused on two principles, parallel feedback and lossy matching [1]–[3]. Either principle has enormous bandwidth potential, ranging from a few megahertz all the way into *Ku*-band. Investigation of the noise in microwave amplifiers with parallel feedback have proven their feasibility for low-noise amplification [4]. When comparing noise figures of feedback amplifiers with those of equivalent amplifiers that use lossy matching circuits, the latter exhibit both, higher theoretical and measured values [5]. However, the lossy match amplifier has the advantage that dc-biasing can be accomplished without seriously reducing the amplifier's bandwidth potential in the megahertz region. A compromise in electrical performance may be found in the combination of both principles.

Employing parallel feedback and/or lossy matching for low-noise applications requires a qualitative study of the influence of all circuit components on the amplifier's noise figure. Several papers on the noise figure of amplifiers with parallel feedback have been published over the last eight years [6]–[9]. However, except for [9] the published results do not take into account the inherent noise sources of the transforming two-ports and therefore cannot be applied to amplifiers that make use of resistive feedback and/or lossy matching networks. This paper develops the exact formulas for the equivalent noise parameters and the noise figure of an amplifier that simultaneously uses parallel feedback and lossy matching while allowing for the thermal noise agitation of all circuit elements. Due to the fact that the results presented here differ from those obtained by applying the formulas presented in [9], a step-by-step account of the deriviations is given in the Appendix.

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II. Noise Figure and Equivalent Noise Parameters

To study the noise of a two-port with internal noise sources, it is replaced by a noise-free two-port preceded by a simple circuit containing its equivalent noise parameters [10]. The latter consists of the equivalent noise resistance R_n, the equivalent noise conductance G_n, and the correlation admittance $Y_{cor} = G_{cor} + jB_{cor}$. The parameters R_n, G_n, and Y_{cor} can be calculated in case the noise figure for optimum noise matching F_{min}, the corresponding signal source admittance $Y_{smin} = G_{smin} + jB_{smin}$, and one other noise figure F and its corresponding signal source admittance $Y_s = G_s + jB_s$, preferably $Y_s = Z_0^{-1}$, are known.

The noise figure of a two-port can be expressed by the well-known formulas [10], [11]

$$F = F_{min} + \frac{R_n}{G_s}(G_s - G_{smin})^2 + \frac{R_n}{G_s}(B_s - B_{smin})^2 \tag{1}$$

with

$$F_{min} = 1 + 2\left[R_nG_{cor} + \sqrt{R_nG_n + (R_nG_{cor})^2}\right]. \tag{2}$$

The circuit whose overall noise figure we want to determine is shown in Fig. 1(a). It consists of a noisy two-port at temperature T embedded in a π-shaped network of three admittances Y_G, Y_{FB}, and Y_D. They contain the conductances G_G, G_{FB}, and G_D which inject noise into the overall two-port of Fig. 1(a) and thereby contribute to the noise figure of the overall network.

In Fig. 1(b) all internal noise sources of the embedded two-port and the surrounding admittances have been extracted and are represented as external noise voltages (v_1, v_{FB}) and noise currents (i_1, i_G, i_D). This step puts all circuit elements and the embedded two-port at $T = 0$ K.

The network of Fig. 1(b) will now be used to determine the noise parameters as shown in Fig. 1(c). In doing so, we follow the procedure as outlined in [4] based on [10]. The admittance matrix of the noiseless network of Fig. 1(c) representing the signal voltages and currents takes the form

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